Math Milestones™ — Grade 6

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:2 **(1)** Would you prefer 33% of a \$100 prize or 75% of a \$50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a \$16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a \$100 item adds ___ dollars to the cost. A 3% tax on a \$1 item adds ___ dollars to the cost.

6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	−4	−42
9:00 pm	−3	−42
10:00 pm	−2	−41
11:00 pm	−1	−40
Midnight	0	−39
1:00 am	1	−39
2:00 am	2	−38

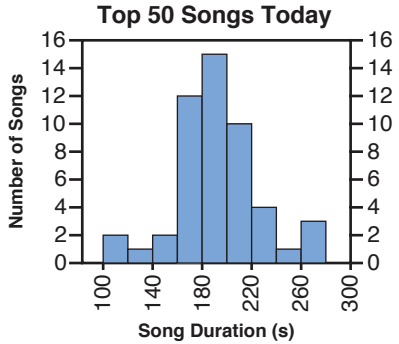
Plot the data on graph paper and label the plot. Describe any patterns you see.

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3) (a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:5 **(1)** Which of the numbers 5, −7, $\frac{2}{3}$, $-\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:7 **(1)** Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. **(2)** Write a sentence about the data giving a measure of center and a measure of variability. **(3)** Make a histogram of the data.* **(4)** Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. **(5)** Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?



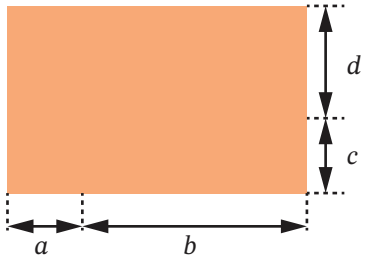
*Use this histogram for (4) and (5) if you don't do (3).

6:8 *Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r - r$?

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Math Milestones™ Task List — Grade 6

The 14 Math Milestones™ tasks for grade 6 have been carefully crafted to embody grade 6 mathematics on one page.

6:1	Charging Cord	C A	6.NS.A.1, 6.EE.B.7
6:2	Prizes, Prices, and Percents	C P	6.RP.A.3c
6:3	South Pole Temperatures	C A	6.NS.C.7, 8
6:4	Gas Mileage	C A	6.RP.A.2, 3
6:5	Positive and Negative Numbers	C	6.NS.C.6, 7
6:6	Planting Corn	C	6.RP.A, 6.EE.C.9
6:7	Song Length Distribution	C P A	6.SP
6:8	Evaluating an Expression	P	6.EE.A
6:9	Truckload of Gravel	C A	6.NS.A.1, 6.EE.B.7
6:10	Weekdays and Weekend Days	C	6.RP.A.1
6:11	Area Expressions	C	6.EE.A
6:12	Coordinate Triangle	C P	6.G.A.1, 3
6:13	Is There a Solution? (Multiplication)	C P	6.EE.B.5
6:14	Dividing Decimals and Fractions	P	6.NS.A.1, 6.NS.B

C = Task has a conceptual focus.
P = Task has a procedural skill & fluency focus.
A = Task has an application focus.

Standards for Mathematical Practice

MP.1	Make sense of problems and persevere in solving them.	6:4, 6:13
MP.2	Reason abstractly and quantitatively.	6:1, 6:2–4 6:6, 6:9–11
MP.3	Construct viable arguments and critique the reasoning of others.	6:5, 6:7, 6:11, 6:13
MP.4	Model with mathematics.	6:2, 6:4, 6:6, 6:7
MP.5	Use appropriate tools strategically.	6:1, 6:4, 6:7, 6:9
MP.6	Attend to precision.	6:4, 6:8, 6:10, 6:14
MP.7	Look for and make use of structure.	6:2, 6:4, 6:5(3), 6:8, 6:11–13
MP.8	Express regularity in repeated reasoning.	6:4, 6:5(1), 6:6

Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

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Math Milestones™ — Grade 7

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

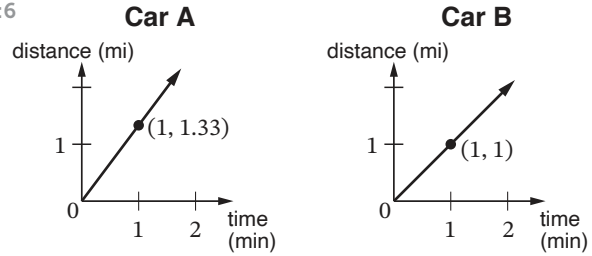
7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

7:4 *"Foul Play."* The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot. His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 one-point free throws. The defender chose to foul and later wondered if it was a good choice. **(1)** To analyze the defender's choice, let's assume that for the Pistons player, every 3-point shot has probability $\frac{1}{3}$ of going in, and every free throw has probability 90% of going in. **(a)** If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. **(2)** Write a paragraph arguing for or against the defender's choice, based on probability calculations and/or simulations.

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of \$7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	**Liba**
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

Math Milestones™ Task List — Grade 7

The 14 Math Milestones™ tasks for grade 7 have been carefully crafted to embody grade 7 mathematics on one page.

7:1	Phone Cost		C P A	7.RP.A.3, 7.EE.A
7:2	Utility Pole Scale Drawing		A	7.G.A.1, 7.G.B.4
7:3	Writing Sums as Products		C P	7.EE.A.1
7:4	"Foul Play"		C A	7.SP.C
7:5	Is There a Solution? (Addition)	🚫	C	7.NS.A.1, 7.EE.B.4
7:6	Car A and Car B		C A	7.RP.A.2
7:7	Speed Limit		A	7.RP.A.1
7:8	Oil Business		A	7.RP.A.2b, 7.EE.A.2, 7.EE.B.4
7:9	Calculating with Rational Numbers	🚫	C P	7.NS.A
7:10	Triangle Conditions		C	7.G.A.2
7:11	Ticket Offers		C A	7.RP.A.3, 7.EE.B
7:12	Temperature Change		C A	7.RP.A.1, 7.NS.A
7:13	Wire Circle		A	7.EE.B.4
7:14	Comparing Rose's and Liba's Solutions		C	7.EE.B.4

C = Task has a conceptual focus. P = Task has a procedural skill & fluency focus. A = Task has an application focus. 🚫 = Task is not designed for use with calculators or other technology.

Standards for Mathematical Practice

MP.1	Make sense of problems and persevere in solving them.	7:3, 7:5, 7:8
MP.2	Reason abstractly and quantitatively.	7:1, 7:6, 7:8, 7:11, 7:12, 7:14
MP.3	Construct viable arguments and critique the reasoning of others.	7:5, 7:10, 7:11
MP.4	Model with mathematics.	7:2, 7:4, 7:6, 7:8, 7:12, 7:13
MP.5	Use appropriate tools strategically.	7:2, 7:4, 7:7, 7:8, 7:10, 7:13
MP.6	Attend to precision.	7:2, 7:7, 7:9, 7:10
MP.7	Look for and make use of structure.	7:1, 7:3, 7:5, 7:8, 7:9, 7:14
MP.8	Express regularity in repeated reasoning.	7:1, 7:11

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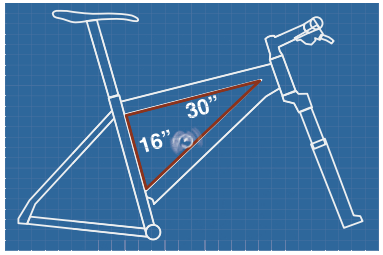
Math Milestones™ — Grade 8

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

8:3 On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.



8:4 **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

$$\begin{cases} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{cases} \quad \begin{cases} d = 100 - 4t \\ d = 3.5 + t \end{cases} \quad \begin{cases} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{cases}$$

8:5 Using physical models, transparencies, or geometry software, illustrate the fact that *rotations take angles to angles of the same measure.*

8:6 Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:8 A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. **(1)** Use technology to plot the data from both days. (View heart rates in a window from 145 to 175.) Describe the main patterns you see. **(2)** On one of the days, the exercise room was warm, and on the other day, the room was cool. Which day do you think was the warm day? Tell how you decided, and support your answer with calculations.

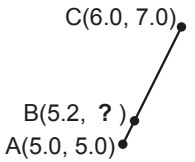
Heart Rate & Effort in Exercise

Day 1 HR, Effort	Day 2 HR, Effort
150.9, 1.3	148.6, 1.6
155.2, 1.5	152.7, 1.9
158.5, 1.8	153.9, 2.3
159.4, 2.1	155.4, 2.9
161.2, 2.1	156.6, 2.9
162.2, 2.3	157.9, 3.1
163.5, 2.4	158.9, 3.6
163.5, 2.7	159.7, 3.7
164.8, 2.7	160.6, 4.1
166.3, 2.9	161.3, 4.2
167.2, 3.0	162.3, 4.3
167.2, 3.3	162.4, 4.6
168.1, 3.4	163.4, 4.7
169.2, 3.4	164.2, 4.8
169.2, 3.5	164.8, 4.7
170.3, 3.5	165.0, 5.0
170.8, 3.6	165.4, 5.1
170.4, 3.7	167.0, 5.2
171.9, 3.7	166.5, 5.3
172.3, 3.9	166.7, 5.4

Click here to get the data online. Each person's heart rate (beats per min.) and effort (1–6 scale) were recorded every 3 min. A group average was then calculated, creating one data point such as (150.9, 1.3).

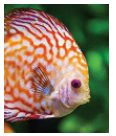
8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.



8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

8:12 Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m^3 of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.



Math Milestones™ Task List — Grade 8

The 12 Math Milestones™ tasks for grade 8 have been carefully crafted to embody grade 8 mathematics on one page.

8:1	Xavier's Notes		A	8.F.B.4
8:2	Pottery Factory		P A	8.EE.C.7b
8:3	Bicycle Blueprint		A	8.G.B.7
8:4	System Solutions	⊘	C P	8.EE.C.8
8:5	Rotations Preserve Angle Measure		C	8.G.A.1
8:6	Rational Form	⊘	C	8.NS.A.1, 8.EE.A.1
8:7	Flight Times and Distances		C A	8.F
8:8	Heart Rate and Exercise		A	8.SP.A.1–3
8:9	Water Evaporation Model		C P A	8.F
8:10	Missing Coordinate		C	8.EE.B
8:11	Angle-Angle Similarity Proof		C	8.G.A.5
8:12	Fish Tank Design		A	8.G.C.9

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Standards for Mathematical Practice

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MP.2	Reason abstractly and quantitatively.	8:1, 8:2, 8:7, 8:9
MP.3	Construct viable arguments and critique the reasoning of others.	8:5, 8:8, 8:11
MP.4	Model with mathematics.	8:1–3, 8:7–9, 8:12
MP.5	Use appropriate tools strategically.	8:1, 8:2, 8:4, 8:5, 8:8, 8:12
MP.6	Attend to precision.	8:1, 8:3, 8:6
MP.7	Look for and make use of structure.	8:4–7, 8:9–11
MP.8	Express regularity in repeated reasoning.	8:2, 8:7

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